KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD–32

PRACTICE PAPER 05 (2023-24)

CHAPTER 05 ARITHMETIC PROGRESSION

(ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**

CLASS : X | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. In an AP, if a = 3.5, d = 0, n = 101, then a_n will be

(a) 0 (b) 3.5 (c) 103.5 (d) 104.5

Ans: (b) $a_{101} = 3.5 + 0\,(100) = 3.5$

2. If p – 1, p + 3, 3p – 1 are in AP, then p is equal to ________.

(a) 3 (b) 4 (c) 2 (d) none of these

Ans: ∵ p – 1, p + 3 and 3p – 1 are in AP.

$\therefore 2(p + 3) = p - 1 + 3p - 1$

$\Rightarrow 2p + 6 = 4p - 2.$

$\Rightarrow -2p = -8 \Rightarrow p = 4.$

3. In an AP, if d = –2, n = 5 and $a_n = 0$, the value of a is

(a) 10 (b) 5 (c) –8 (d) 8

Ans: (d) d = – 2, n = 5, $a_n = 0$

$\because a_n = 0$

$\Rightarrow a + (n - 1)d = 0$

$\Rightarrow a + (5 - 1)(-2) = 0$

$\Rightarrow a = 8$

Correct option is (d).

4. If the common difference of an AP is 3, then $a_{20} - a_{15}$ is

(a) 5 (b) 3 (c) 15 (d) 20

Ans: (c) Common difference, d = 3

$a_{20} - a_{15} = (a + 19d) - (a + 14d) = 5d = 5 \times 3 = 15$

5. The next term of the AP $\sqrt{18}, \sqrt{50}, \sqrt{98}, ...$ is

(a) $\sqrt{146}$ (b) $\sqrt{128}$ (c) $\sqrt{162}$ (d) $\sqrt{200}$

Ans: (c) $\sqrt{18}, \sqrt{50}, \sqrt{98}, ... = 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, ...$

Next term is $9\sqrt{2} = \sqrt{162}$

6. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, ...$ is

(a) p (b) – p (c) –1 (d) 1

Ans: (c) Common difference = $a_2 - a_1 = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = -1$

7. An AP consists of 31 terms. If its 16th term is m, then sum of all the terms of this AP is
(a) 16 m (b) 47 m (c) 31 m (d) 52 m
Ans: (c) $S_{31} = \frac{31}{2}(2a + 30d)$
$a_{16} = a + 15d = m$
$\Rightarrow S_{31} = \frac{31}{2} \times 2(a + 15d) \Rightarrow S_{31} = 31m$

8. If the sum of first n terms of an AP is $An + Bn^2$ where A and B are constants, the common difference of AP will be
(a) A + B (b) A – B (c) 2A (d) 2B
Ans: (d) $S_n = A_n + Bn^2$
$S_1 = A \times 1 + B \times 1^2 = A + B$
$\because S_1 = a_1$
$\therefore a_1 = A + B$ ... (i)
and $S_1 = A \times 2 + B \times 2^2$
$\Rightarrow a_1 + a_2 = 2A + 4B$
$\Rightarrow (A + B) + a_2 = 2A + 4B$ [Using (i)]
$\Rightarrow a_2 = A + 3B$
$\therefore d = a_2 - a_1 = 2B$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a)Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b)Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c)Assertion (A) is true but reason (R) is false.
(d)Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The sum of series with the nth term an= (9 – 5n) is 220 when no. of terms n = 6
Reason (R): Sum of first n terms in an A.P. is given by the formula: $S_n = \frac{n}{2}[2a + (n-1)d]$
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (A):** The value of n, if a = 10, d = 5, an = 95 is 20
Reason (R): The formula of general term an is an = a + (n - 1)d.
Ans: (d) Assertion (A) is false but reason (R) is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Determine k so that $4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$ are three consecutive terms of an AP.
Ans: For consecutive terms of AP,
$2(2k^2 + 3k + 6) = (3k^2 + 4k + 4) + (4k + 8)$
$\Rightarrow 4k^2 + 6k + 12 = 3k^2 + 8k + 12 \Rightarrow k^2 - 2k = 0$
$\Rightarrow k(k - 2) = 0 \Rightarrow k = 0$ or $k = 2$

12. In an AP, the 24th term is twice the 10th term. Prove that the 36th term is twice the 16th term.
Ans: Let 1st term = a, common difference = d.
$a_{10} = a + 9d$, $a_{24} = a + 23d$

According to the question, $a_{24} = 2 \times a_{10}$
$\Rightarrow a + 23d = 2(a + 9d) \Rightarrow a + 23d = 2a + 18d \Rightarrow a = 5d$
Now, $a_{16} = a + 15d = 5d + 15d = 20d$ ...(i)
$a_{36} = a + 35d = 5d + 35d = 40d$ ...(ii)
From (i) and (ii), we get
$a_{36} = 2 \times a_{16}$ Hence proved.

13. Find 10th term from end of the AP 4, 9, 14, ... , 254.
Ans: 10th term from end of AP 4, 9, 14, ..., 254 is 10th term of the AP 254, 249, 244, ... 14, 9, 4
Here $a = 254$
$d = 249 - 254 = -5$
$\therefore\ a_{10} = a + 9d \Rightarrow a_{10} = 254 + 9 \times (-5) = 209$

14. If the sum of first n terms of an AP is given by $S_n = 3n^2 + 2n$, find the nth term of the AP.
Ans: Given: $S_n = 3n^2 + 2n$. Let a_n be the nth term.
$\therefore\ a_n = S_n - S_{n-1} = (3n^2 + 2n) - \{3(n-1)^2 + 2(n-1)\} = 3n^2 + 2n - 3n^2 + 6n - 3 - 2n + 2$
Hence, $a_n = 6n - 1$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the value of the middle term of the following AP: –6, –2, 2, ..., 58.
Ans: Here, $a = -6$, $d = -2 + 6 = 4$ and $a_n = 58$
$a_n = 58$
$\Rightarrow a + (n-1)d = 58 \Rightarrow -6 + (n-1)4 = 58$
$\Rightarrow (n-1)4 = 64 \Rightarrow n - 1 = 16 \Rightarrow n = 17$ (odd)
$\therefore$ Middle term = $\dfrac{17+1}{2} = \dfrac{18}{2}$ = 9th term
$\therefore$ 9th term is the middle term.
Now, $a_9 = a + 8d = -6 + 8 \times 4 = -6 + 32 = 26$

16. Which term of the progression 19, $18\frac{1}{5}, 17\frac{2}{5}, \ldots\ldots$ is the first negative term.
Ans: The given expression is 19, $18\frac{1}{5}, 17\frac{2}{5}, \ldots$
Here, $a_2 - a_1 = \dfrac{91}{5} - 19 = \dfrac{91-95}{5} = \dfrac{-4}{5}$
$a_3 - a_2 = \dfrac{87}{5} - \dfrac{91}{5} = \dfrac{-4}{5}$
Therefore, (i) is an AP with $a = 19$, $d = \dfrac{-4}{5}$
Let the nth term of the given AP be the first negative term. Then, nth term < 0.
$\Rightarrow T_n < 0 \Rightarrow 19 + (n-1)\dfrac{-4}{5} < 0$
$\Rightarrow (99 - 4n) < 0 \Rightarrow 4n > 99 \Rightarrow n > 24\frac{3}{4}$
$\therefore\ n = 25$, i.e. 25th term is the first negative term in the given AP.

17. If the pth, qth, rth terms of an AP be x, y, z respectively, show that $x(q - r) + y(r - p) + z(p - q) = 0$.
Ans: Let a be the first term and d be the common difference of the AP.
$a_p = x \Rightarrow a + (p - 1)d = x$ ...(i)

$a_q = y \Rightarrow a + (q - 1)\, d = y$ ...(ii)
$a_r = z \Rightarrow a + (r - 1)\, d = z$ ...(iii)
Substituting the values of x, y and z from (i), (ii) and (iii), we get
$x(q - r) + y(r - p) + z(p - q)$
$= [a + (p - 1)d]\,(q - r) + [a + (q - 1)d]\,(r - p) + [a + (r - 1)d]\,(p - q)$
$= a[(q - r) + (r - p) + (p - q)] + d\,[(p - 1)\,(q - r) + (q - 1)(r - p) + (r - 1)\,(p - q)]$
$= a(0) + d\,[p(q - r) + q\,(r - p) + r\,(p - q) - (q - r + r - p + p - q)] = d(0 - 0) = 0.$

SECTION – D

Questions 18 carry 5 marks.

18. If S_1, S_2, S_3 are the sum of n terms of three APs, the first term of each being unity and the respective common difference being 1, 2, 3; prove that $S_1 + S_3 = 2S_2$.
Ans: Here $d_1 = 1$, $d_2 = 2$, $d_3 = 3$
Ist term, $a = 1$

$$S_1 = \frac{n}{2}\,[2 \times 1 + (n - 1)d_1] = \frac{n}{2}\,[2 + (n - 1)1] = \frac{n}{2}\,(n + 1)$$

$$S_2 = \frac{n}{2}\,[2 \times 1 + (n - 1)d_2] = \frac{n}{2}\,[2 + (n - 1)2] = n^2$$

$$S_3 = \frac{n}{2}\,[2 \times 1 + (n - 1)3] = \frac{n}{2}\,[3n - 1]$$

Now $S_1 + S_3 = \frac{n}{2}\,(n + 1) + \frac{n}{2}\,(3n - 1) = \frac{n}{2}\,(n + 1 + 3n - 1) = \frac{n}{2} \times 4n = 2n^2$ ...(i)
and $2 \times S_2 = 2 \times n^2$ ...(ii)
From (i) and (ii), we have
$S_1 + S_3 = 2S_2$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Aditya is a fitness freak and great athlete. He always wants to make his nation proud by winning medals and prizes in the athletic activities.

An upcoming activity for athletes was going to be organised by Railways. Aditya wants to participate in 200 m race. He can currently run that distance in 51 seconds. But he wants to increase his speed, so to do it in 31 seconds. With each day of practice, it takes him 2 seconds less.
(i) He wants to makes his best time as 31 sec. In how many days will be able to achieve his target? (2)
(ii) What will be the difference between the time taken on 5th day and 7th day. (2)
OR
(ii) Which term of the arithmetic progression 3, 15, 27, 39 .... will be 120 more than its 21st term? (2)

Ans: Ans:
(i) Let, the number of days taken to achieve the target be n.
In the given A.P., a = 51, d = – 2
Since an = a + (n – 1)d ⇒ 31 = 51 + (n – 1)(– 2)
⇒ 31 – 51 = (n – 1)(– 2) ⇒ – 20 = (n – 1) (– 2)
⇒ (n – 1) = 10 ⇒ n = 11
Hence, 11 days are needed to achieve the target.
(ii) a_5 = a + 4d = 51 + 4(– 2) = 51 – 8 = 43 sec
a_7 = a + 6d = 51 + 6(– 2) = 51 – 12 = 39 sec
Now, time difference = 43 – 39 = 4 sec.

OR

(ii) We have, a = 3 and d = 12
∴ a_{21} = a + 20d = 3 + 20 × 12 = 243
Let nth term of the given AP be 120 more than its 21st term. Then, a_n = 120 + a_{21}
⇒ 3 + (n – 1)d = 120 + 243
⇒ 3 + 12(n – 1) = 363 ⇒ 12(n – 1) = 360
⇒ n – 1 = 30 ⇒ n = 31
Hence, 31st term of the given AP is 120 more than its 21st term.

20. In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021–22, as per a report. A car manufacturing company planned to produce 1800 cars in 4th year and 2600 cars in 8th year. Assuming that the production increases uniformly by a fixed number every year.

Based on the above information answer the following questions.
(i) Find the production in the 1st year. (1)
(ii) Find the production in the 12th year. (1)
(iii) Find the total production in first 10 years. (2)

OR

(iii) In how many years will the total production reach 31200 cars? (2)
Ans: (i) Since the production increases uniformly by a fixed number every year, the number of Cars manufactured in 1st, 2nd, 3rd, . . .,years will form an AP.
So, a + 3d = 1800 & a + 7d = 2600
So d = 200 & a = 1200
(ii) a_{12} = a + 11d ⇒ a_{30} = 1200 + 11 × 200
⇒ a_{12} = 3400
(iii) $S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow S_{10} = \frac{10}{2}[2\times1200+(10-1)\times200]$
$\Rightarrow S_{10} = 5[2400+1800] = 5\times4200 = 21000$

OR

$S_n = \frac{n}{2}[2a+(n-1)d] = 31200$

$\Rightarrow \frac{n}{2}[2\times 1200+(n-1)\times 200]=31200$

$\Rightarrow \frac{n}{2}\times 200[12+(n-1)]=31200$

$\Rightarrow n[12 + (n - 1)] = 312$

$\Rightarrow n^2 + 11n - 312 = 0$

$\Rightarrow n^2 + 24n - 13n - 312 = 0$

$\Rightarrow (n + 24)(n - 13) = 0$

$\Rightarrow n = 13$ or -24.

As n can't be negative. So n = 13